Judgment Aggregation: June 2018 Project

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Plan for Today

- Some words about the homework exercise
- Ways to circumvent the impossibility result of List and Pettit:
  - Domain restrictions.
  - Relaxing our axioms.
  - Agenda properties (Wednesday)
- Axiomatic Characterisation of a class of rules
  - Quota Rules (and specifically Majority rule)

Impossibility from Yesterday

Theorem 1 (List and Pettit, 2002) For $n \geqslant 2$, No judgment aggregation rule for an agenda Φ with $\{p, q, p \wedge q\} \subseteq \Phi$ satisfies anonymity, neutrality, independence, completeness and consistency.

Which did the majority rule fail?

C. List and P. Pettit. Aggregating Sets of Judgments: An Impossibility Result. *Economics and Philosophy*, 18(1), 89-110, 2002.

Ways out of the Impossibility

We will look at two ways to avoid the Impossibility.

- Domain Restrictions (concerns input to F)
- Relaxing the Axioms (concerns output of F)

1) Domain Restrictions

Recall: $F : \mathcal{J}(\Phi)^n \to 2^{\Phi}$. Where $\mathcal{J}(\Phi)^n$ is all n-agent profiles made up of complete and consistent judgment sets.

When we restrict the domain of an aggregation rule, this means that we look at functions with domain $\mathcal{X} \subset \mathcal{J}(\Phi)^n$.

Note that a domain restriction does not mean we allow incomplete or inconsistent judgment sets. We are restricting which profiles in $\mathcal{J}(\Phi)^n$ are allowed.

Quick! The dumbest domain restriction you can think of? 🌟

Unidimentional Alignment

A profile is unidimentionally aligned if we can order the agents such that for each (positive) proposition $p \in \Phi$, the agents accepting p are either all to the left or all to the right of the agents rejecting p.

	1	2	3	4	5
p	✓	✓	×	×	×
q	×	×	×	×	✓
$p \to q$	×	×	✓	✓	✓

Theorem 2 (List, 2003) For any unidimentionally aligned profile, the majority will return a consistent outcome.
Note: If n is odd, we are also guaranteed completeness.

C. List. A Possibility Theorem on Aggregating over Multiple Interconnected Propositions. *Mathematical Social Sciences*, 45(1), 1-13, 2003.

Proof.

We do the proof for odd n.

	1	2	3	4	5	Majority
p	✓	✓	×	×	×	×
q	×	×	×	×	✓	×
$p \to q$	×	×	✓	✓	✓	×

Call agent number $\lceil \frac{n}{2} \rceil$ (according to ordering) the median agent.

- For each $\varphi \in \Phi$, at least $\lceil \frac{n}{2} \rceil$ agents accept φ if the median agent does.
  - $\varphi \in J_{\lceil \frac{n}{2} \rceil} \Rightarrow \varphi \in F_{Maj}(\boldsymbol{J})$
  -
  - $\varphi \notin J_{\lceil \frac{n}{2} \rceil} \Rightarrow \varphi \notin F_{Maj}(\boldsymbol{J})$
- Since the median agent submits a consistent judgment set, the outcome of the (strict) majority will be consistent. 👍

Value Restriction

A set $S \subseteq \Phi$ is minimally inconsistent if it is inconsistent, and every $X \subset S$ is consistent.

A profile $\boldsymbol{J}$ is value-restricted if for every mi-set $S \subseteq \Phi$, there are distinct $\varphi, \psi \in S$ such that no agent i has a judgment set where $\{\varphi, \psi\} \subseteq J_i$.

Theorem 2 (Dietrich and List, 2010) For any value-restricted profile, the majority will return a consistent outcome.

F. Dietrich and C. List. Majority Voting on Restricted Domains. *Journal of Economic Theory*, 145(2), 512-543, 2010.

Proof.

Let $\boldsymbol{J}$ be a value-restricted profile. Assume for contradiction that $F_{Maj}(\boldsymbol{J})$ is inconsistent. Then there exists a set $S \subseteq F_{Maj}(\boldsymbol{J})$ that is minimally inconsistent.

Since $\boldsymbol{J}$ is value-restricted, we know there are two formulas $\varphi, \psi \in S$ such that no agent accepts both formulas.

But since $\varphi, \psi \in S$ and $S \subseteq F_{Maj}(\boldsymbol{J})$, there must have been a strict majority for each of φ and ψ.

Thus, there must be at least one agent who accepted both formulas. Which contradicts our assumption that $\boldsymbol{J}$ is value restricted! 👍

Interpretation

Unidimentional Alignment:

- We can interpret the left to right ordering of the individuals as their location on some ideological dimension.
- Each proposition is somewhere on the left-right spectrum and individuals are located somewhere on this spectrum
- Ex: political issues

Value-restriction:

- A (weaker) and more abstract restriction that is implied by several more "intuitive" ones (including UA).

2) Relaxing Axioms

We already discussed a bit yesterday how it might not be terrible if a rule does not satisfy all our axioms.

We'll see a couple examples of rules which return complete and consistent collective judgments at the expense of one (or more) of the axioms of the impossibility.

Which axiom do you think is a good candidate to relax? 🌟
What would a rule that is not anonymous (but still independent and neutral) look like? 🌟

Premise-based Rule(s)

Premise based rules: divide the agenda into premises—Φ_P—and conclusions—Φ_C. Aggregate opinion on premises, then accept a conclusion C if accepted premises imply C.

$$F_{Pre} = \Delta \cup \{\varphi \in \Phi_C \mid \Delta \models \varphi\}$$

Fails Independence (& Neutrality), but if Φ_P is the set of literals and the agenda is closed under propositional letters, then for odd n, $F_{Pre}(\boldsymbol{J})$ is consistent and complete.

Distance-based Rule(s)

Kemeny Rule: $F(\boldsymbol{J}) = \underset{J \in \mathcal{J}(\Phi)}{\text{argmin}} \sum_{i \in \mathcal{N}} H(J, J_i)$

Where $H(J, J') = |J \setminus J'|$ is the Hamming distance.

Kemeny chooses the judgment set which minimises the sum of (Hamming) distances to the judgment sets in the profile.

- Fails Independence (and Neutrality).
- Guarantees consistency by definition.
- Computationally hard to determine outcome!

U. Endriss, U.Grandi and D.Porello. Complexity of Judgment Aggregation. Journal of Artificial Intelligence Research, 45, 481-514, 2012.

Which do you Find More Convincing?

Formally Defining the Majority Rule

Recall: $N_\varphi^{\boldsymbol{J}}$ is the set of agents who accept φ in profile $\boldsymbol{J}$

The (strict) majority rule F_{Maj} takes a profile (of complete & consistent judgment sets) and returns the $\varphi \in \Phi$ that are accepted by more than half the agents.

$$F_{Maj}(\boldsymbol{J}) = \{\varphi \in \Phi \mid |N_\varphi^{\boldsymbol{J}}| > \frac{n}{2}\}$$

Yesterday we saw that F_{Maj} is Independent, Anonymous, Neutral and Complement-free. For odd n, it is also Complete.

Quota Rules

We define a quota rule by a function $q : \Phi \to \{0, \ldots, n+1\}$.

$$F_q(\boldsymbol{J}) = \{\varphi \mid |N_\varphi^{\boldsymbol{J}}| \geqslant q(\varphi)\}$$

A quota rule is uniform if $q(\varphi)$ is the same for all $\varphi \in \Phi$.

The (strict) majority rule is uniform quota rule with q $= \lfloor \frac{n}{2} \rfloor + 1$.

If a quota rule is not uniform, which axiom is violated?

Axiomatic Characterisations

F satisfies some conditions $\Leftrightarrow$ F is in a certain class of rules.

- Tells us that if we want certain axioms satisfied, we have to look in that class.
- And, if we are using a rule from the class, we can be sure those axioms are satisfied.

Note: Axiomatic characterisations do not capture all properties. There may be more than one way of characterising a rule.

A New Axiom

Note: $\boldsymbol{J} =_{-i} \boldsymbol{J'}$ means for all agents $j \neq i$, $J_j = J'_j$.

- Anonymity: for any profile $\boldsymbol{J}$ and any permutation $\pi : \mathcal{N} \to \mathcal{N}$, we have that $F(J_1, \dots J_n) = F(J_{\pi(1)}, \dots J_{\pi(n)})$.
- Neutrality: for any $\varphi, \psi \in \Phi$ and any profile $\boldsymbol{J}$, if $\varphi \in J_i \Leftrightarrow \psi \in J_i$ for all $i \in \mathcal{N}$, then $\Phi \in F(\boldsymbol{J}) \Leftrightarrow \in F(\boldsymbol{J})$.
- Independence: for any $\varphi \in \Phi$ and any two profiles $\boldsymbol{J}$ and $\boldsymbol{J'}$, if $\varphi \in J_i \Leftrightarrow \varphi \in J'_i$ for all $i \in \mathcal{N}$, then $\varphi \in F(\boldsymbol{J}) \Leftrightarrow F(\boldsymbol{J'})$.
- Monotonicity: Additional support should not "harm".
  - for any $\varphi \in \Phi$ and profiles $\boldsymbol{J}$ and $\boldsymbol{J'}$, $\boldsymbol{J} =_{-i} \boldsymbol{J'}$, and $\varphi \in J'_i \setminus J_i$ for some agent $i \in \mathcal{N}$, then $\varphi \in F(\boldsymbol{J}) \Rightarrow \varphi \in F(\boldsymbol{J'})$.

Characterisation of Quota Rules

Theorem 2 (Dietrich and List, 2007). An aggregation rule F is anonymous, independent and monotonic iff it is a quota rule.

Proof.

- ⇐ Clear from the definition of a quota rule.
- ⇒ By independence, we decide formula by formula. By anonymity, only the size of the coalitions matter. By monotonicity if a set of agents can get φ accepted, then a superset of those can also get φ accepted. This means that for every formula φ, there is some number k such that φ is accepted if and only if at least k agents accept φ. I.e. $k = q(\varphi)$.

A quota rule is neutral if and only if it is a uniform quota rule.

Corollary 1. F is ANIM iff it is a uniform quota rule .

Dietrich, F. and List, C. Judgment Aggregation by Quota Rules: Majority Voting Generalized. *Journal of Theoretical Politics*, 19(4), 391-424, 2007.

Characterisation of Majority Rule

Corollary 2. for odd n: F is ANIM, complete and complement-free if and only if F is the (strict) majority rule.

Proof Sketch:

- Majority is a uniform quota rule, so we get ANIM for free.
- If q is high we get complement-freeness. If q is low, we get completeness. The majority rule hits the sweet spot.

Note: For even n, no rule satisfies ANIM + C & C.

Last Slide

Summary:

- In the first part of the lecture we saw two different ways of circumventing the impossibility of List and Pettit.
- In the second part we saw characterisation results for (uniform) quota rules in general, and the majority rule.

Homework & Next Lecture:

- The homework is up on the website. Deadline 11am.
- Some notes on the presentations will be up today.
- Tomorrow: Zoi will take over and talk about the agenda!